# Time Travel

*A. Types of time travel*

1) Funny world lines in normal spacetime

2) Normal world lines in a spacetime with funny causal structure

*B. Paradoxes of time travel*

What does the "grandfather paradox" and other paradoxes of time travel show?

1) That time travel is conceptually incoherent? That it is not physically possible?

How can it show any such thing since there are mathematically consistent models of time travel (in the sense of CTCs) that satisfy Einstein's field equations, energy conditions, and other plausible conditions for physical possibility.

2) Time travel clashes with free will.

Possibly. But free will is a notoriously slippery and problematic concept, even without time travel thrown into the mix.

Time travel paradoxes can be formulated in terms of inanimate objects. Best to investigate these forms of the paradoxes which do not implicate the puzzles of free will.

**Thesis**: The paradoxes of time travel are a crude way of illustrating a disconnect between two senses of physical possibility that occurs in spacetimes with CTCs.

Assume that the basic laws of physics are *local* in the sense that if $M, G_1, G_2, \ldots, G_n$ satisfies the laws then for any open neighborhood $U \subset M$, $U, G_1|_U, G_2|_U, \ldots, G_n|_U$.

*Def.* A local state of affairs $S(U)$, $U \subset M$, is physically possible$_1$ iff $S(U)$ is compatible with the laws.

*Def.* A local state affairs $S(U)$, $U \subset M$, is physically possible$_2$ iff there is a global state of $S(M)$ compatible with the laws such that $S(M)|_U = S(U)$.

In normal spacetimes a PP$_1$ state is also PP$_2$. But in spacetimes with CTCs a state that is PP$_1$ can fail to be PP$_2$.

Examples

Everett, “Time travel paradoxes, path integrals, and the many worlds interpretation of quantum mechanics,” *Physical Review D* **69** (2004): 124023

“CTCs lead to well-known problems with paradoxes arising from the apparent possibility of inconsistent causal loops. This phenomenon is illustrated by the ‘grandfather paradox’ …

Satisfactory physical theories must avoid giving rise to such self-contradictory predictions. One approach to achieving this is to impose consistency constraints on the allowable initial conditions on spacelike surfaces prior to the formation of the CTCs, thus abandoning the principle that initial conditions on such surfaces can be chosen at will [sic!]. For example, in the case of the grandfather paradox we might insist that the initial conditions just before the prospective murder include the presence of a strategically placed banana peel on which the prospective murder slips as he pulls the trigger, thus spoiling his aim. One might refer to this approach as the ‘banana peel mechanism’; it leads to a theory free of logical contradictions, but requires occurrences that would seem, *a priori*, to be highly improbable. This violates strong intuitive feelings. These feeling may simply reflect our lack of experience involving CTCs [indeed!] Nevertheless, a need to invoke constraints on initial conditions would be quite disturbing for many physicists and contribute to an expectation that CTCs are forbidden.”

*Note*: It is NOT untypical for relativistic field equations to imply constraints on initial data.

- Maxwell's equations of EM imply that the **E** and **B** fields cannot be freely specified at a given time.

- Einstein's gravitational field equations imply that the metric field at a given time cannot be freely specified.

Call these the *basic constraints*, the constraints that must be met for $PP_1$.

In spacetimes with CTCs, additional constraints may be needed to guarantee $PP_2$. Call these the *CTC constraints*.

The paradoxes of time travel are one way to indicate that the CTC constraints can be highly non trivial.

*Question*: Does the apparent absence of such constraints indicate that we do not live in a world with CTCs?

*BIG PROBLEM*: What is the status of the CTC constraints?

If we are operating with a fixed background spacetime (which may contains CTCs) and are considering the propagation of some field on this spacetime, then it seems that the CTC constraints are (derivative) laws since they are implications of the local laws and the fixed structure of spacetime.

If so, we have an easy answer to the grandfather paradox—Kurt can't do certain things because they are incompatible with the laws.

*Complaint*: GTR teaches us that the structure of spacetime is not a "given" but varies from solution to solution. This lesson undermines the law status of the CTC constraints.

But by the same token, what sense can be make of the notion of CTC constraint if we abandon the model of a field propagating on a fixed background spacetime?

## Time Machines

*Time machine*: a device that produces CTCs.

*Issue 1*: What conditions must a spacetime satisfy in order that it is plausible to see a time machine at work?

*Issue 2*: What no-go results have been proved? Do they collectively justify the conclusion that the laws of physics forbid the operation of a time machine?

## *Conditions for a Time Machine*

(T1) There is a time slice (spacelike hypersurface without edges) $\Sigma \subset M, g_{ab}$ such that no CTCs exist to the past of $\Sigma$.

*Comments*: Think of $\Sigma$ as a time shortly before the time machine is turned on. If CTCs already exist, a time machine is not needed to produce them. Note that this excludes Gödel spacetime since it does not possess any time slices.

(T2) In addition assume that $\Sigma$ is achronal and, therefore, a partial Cauchy surface.

*Comment*: Adding (T2) does not lose much generality, since if a time slice $\Sigma$ is not achronal, it can be made so by passing to a covering spacetime.

(T3) The region $TM \subset M$ corresponding to the operation of the time machine is finite, i.e. contained in a compact subset of $M$.

*Comment*: The time machine should be confined to a finite volume of space and should operate for a finite amount of time.

(T4) $TM \subset D^{+}(\Sigma)$.

*Comment*: The operation of the time machine should be determined by the instructions encoded on $\Sigma$.

The key problem is to write a condition that guarantees that the operation of the time machine is responsible for development of CTCs. Here are a couple of attempts to do so.

(T5) Let $V$ be the chronology violating region (i.e. the largest region such that through every $p \in M$ there is a CTC). Require that $V \subset J^{+}(TM)$.

*Comment*: (T5) says that all of $V$ can be causally influenced by the operation of the time machine. But it doesn't assure that $V$ isn't influenced in some decisive way by influences coming from outside $TM$.

(T6) $V \subset D^+(TM)$.

*Comment*: If this condition made sense it would guarantee that the time machine is (or can be) responsible—in the sense of causal determinism—for the formation of CTCs. But it doesn't make sense: not only can $V$ not be contained in $D^+(TM)$, it cannot be contained in $D^+(\Sigma)$ (since $D^+(\Sigma)$ is globally hyperbolic). $V$ must lie beyond $H^+(\Sigma)$.

(T7) Hawking's condition: $H^+(\Sigma)$ is compactly generated.

*Comment*: Hawking's condition rules out that $V$ is affected by influences coming out of singularities or infinity rather than from the time machine. But how does it assure that if CTCs do develop they are due to the time machine? This is not a problem if the goal is to prove no-go results since then (T7) only has to be a necessary condition for the operation of a time machine. But is (T7) a necessary condition?

(T8) Every "suitable" extension of $D^+(\Sigma)$ contains CTCs.

*Comment*: Next-best-thing to requiring that the operation of the time machine causally determines the emergence of CTCs. It guarantees that CTCs do form, although the precise manner in which they form is left open since there may be distinct extensions that contain CTCs. (T8) rules out Politzer spacetime as a time machine spacetime.

What is a “suitable” extension?

- EFE and energy conditions satisfied
- inextendible
- hole free

*Def*. $M, g_{ab}$ is hole free iff for any spacelike $S \subset M$, there is no isometric imbedding $i : S \rightarrow M', g'_{ab}$ into another spacetime such that $i(D(S))$ is a proper subset of $D(i(S))$.

Manchak: If $\Sigma$ is a slice of the Taub portion of Misner spacetime and $\overline{D^+(\Sigma)}$ is hole free, then every smooth extension contains CTCs. So (T8) is not empty.

## *No-Go Results for Time Machines*

*A. Classical GTR*

(1) Hawking's chronology protection theorem.

Is compactly generated Cauchy horizons necessary for a time machine?

(2) Tipler (1976, 1977): if EFE and WEC satisfied then the evolution of CTCs from regular initial data on an asymptotically flat partial Cauchy surface is accompanied by singularities.

Tipler's gloss: a time machine must manufacture CTCs using only ordinary matter—WEC satisfied and matter density does not become arbitrarily great. But (i) the sense of singularity in the theorem is geodesic incompleteness, and this need not entail unbounded matter density, and (ii) if the singularities do not occur before the CTCs form, they cannot be used to complain that CTCs are not being manufactured using ordinary matter.

(3) No-go results using (T8)???

*B. Semi-classical quantum gravity* (SCQG)

(1) A mechanism for chronology protection? Indications that $\langle T_{ab} \rangle$ diverges as the chronology horizon $H^+(\Sigma)$ is crossed, leading to the formation of spacetime singularities which cut off further development.

But counterexamples found.

(2) Kay, Radzikowski, Wald (1997).

If $H^+(\Sigma)$ is compactly generated then $\langle T_{ab} \rangle$ is not well-defined at $H^+(\Sigma)$.

- SCQG breaks down when CTCs try to form. Does this provide a mechanism for chronology protection or just show the limitations of SCQG?

- What about cases where $H^+(\Sigma)$ is not compactly generated?